For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Evaluate:
$$\lim_{x\to\frac{\pi}{4}} \frac{\cos^2(x)-\sin^2(x)}{\cos^4(x)-\sin^4(x)}$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** Does Not Exist **(E)** NOTA

2. If $f(x)=(x-1)^3\cdot(2x-1)^4$, what is $f'(1)$?

(A) 0 **(B)** 1 **(C)** 4 **(D)** 12 **(E)** NOTA

3. If $f'(x)=(x-a)(x-b)$, where does $f(x)$ have a local maximum?

(A) $x=a$ **(B)** $x=b$ **(C)** $x=a$ and $x=b$ **(D)** Need More Info. **(E)** NOTA

4. Which of the following functions are continuous on the interval $(-\infty,\infty)$, but not differentiable on that entire interval?

I. $f(x)=|x-3|+6$ II. $g(x)=\sqrt[3]{x}+3$ III. $h(x)=\sin(6x-3)$ IV. $j(x)=e^{ex}$

(A) I **(B)** I, II **(C)** I, II, IV **(D)** I, III, IV **(E)** NOTA

5. Evaluate:
$$\lim_{n\to\infty}\left(1+\frac{5}{n}\right)^{2n}$$

(A) e **(B)** e^2 **(C)** e^5 **(D)** e^{10} **(E)** NOTA

6. What value(s) satisfy the Mean Value Theorem for Derivatives for the function $f(x)=2x^2-x$ over the interval $[2,4]$?

(A) -3 **(B)** -0.5 **(C)** 3 **(D)** $-3,3$ **(E)** NOTA

7. What theorem is being depicted below?
"If a real-valued function f is continuous on a proper closed interval $[a,b]$, differentiable on the open interval (a,b), and $f(a)=f(b)$, then there exists at least one value c in the open interval (a,b) such that $f'(c)=0$."

(A) Mean Value Theorem for Derivatives **(B)** Mean Value Theorem for Integrals
(C) Rolle's Theorem **(D)** Fundamental Theorem of Calculus **(E)** NOTA

8. Approximate $\int_0^4 x^2\,dx$ using a left-hand Riemann sum with four equal subdivisions.

(A) 14 **(B)** 15 **(C)** 22 **(D)** 30 **(E)** NOTA

9. Approximate $\sqrt{0.99}$ to the thousandths place using the linear approximation for $f(x)=\sqrt{1-x}$ at $x=0$.

(A) 0.990 **(B)** 0.995 **(C)** 1.000 **(D)** 1.005 **(E)** NOTA

10. Find the volume of the solid generated when the region bounded by $f(x) = -\left(x - \frac{5}{2}\right)^2 + \frac{9}{4}$ and the x-axis is revolved $\frac{3\pi}{2}$ radians about the x-axis.

(A) $\frac{81\pi}{34}$ **(B)** $\frac{166\pi}{27}$ **(C)** $\frac{221\pi}{28}$ **(D)** $\frac{243\pi}{40}$ **(E)** NOTA

11. Find the average rate of change of the function $f(x) = 3x^3 + 2x^2 + x + 2016$ on the interval $[3, 5]$.

(A) 328 **(B)** 164 **(C)** 246 **(D)** 94 **(E)** NOTA

12. Find the area bounded by the curves $x - 4y = -6$ and $x = (y-2)^2 + 5$.

(A) $\frac{1}{3}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{8}{3}$ **(D)** $\frac{100}{3}$ **(E)** NOTA

13. If $f(x) = \sin(2x)$, what is $f^{(2016)}(x)$?

(A) $-2^{2016}\cos(2x)$ **(B)** $2^{2016}\cos(2x)$ **(C)** $-2^{2016}\sin(2x)$ **(D)** $2^{2016}\sin(2x)$ **(E)** NOTA

14. Meit is rolling! His path is given by $f(x) = x^3 - 3x^2 + x - 2$. When is he accelerating?

(A) $x < 1$ **(B)** $x > 1$
(C) $0 < x < \frac{3-\sqrt{6}}{3}$ and $1 < x < \frac{3+\sqrt{6}}{3}$ **(D)** $\frac{3-\sqrt{6}}{3} < x < 1$ and $x > \frac{3+\sqrt{6}}{3}$ **(E)** NOTA

15. Aditya has 60 m^2 of cardboard and wants to make a regular right hexagonal prism. What should the side length of the base be, in meters, if Aditya wants to maximize the volume of the prism?

(A) $\frac{4\sqrt{15}}{9}$ **(B)** $\frac{20\sqrt{3}}{9}$ **(C)** $\frac{2\sqrt[4]{75}}{3}$ **(D)** $\frac{4\sqrt[4]{60}}{3}$ **(E)** NOTA

16. Find the y-coordinate of the point on $y = \sqrt{x}$ that is closest to $(2, 1)$.

(A) -1 **(B)** 0 **(C)** $\frac{1+\sqrt{3}}{2}$ **(D)** $\frac{2+\sqrt{3}}{2}$ **(E)** NOTA

17. Kyle is at the beach and decides to make a sandcastle. He pulls out a bucket that is a perfect circular frustum with a height of 5 m and radii of 4 m and 14 m. He puts the empty bucket down so that the smaller circular base is on the bottom and uniformly pours sand in at a rate of 3 m^3/s. What is the rate of change in m^3/s of the upper radius of the sand when the height of the sand is 3 m?

(A) $\frac{3}{50\pi}$ **(B)** $\frac{\sqrt{3\pi}}{2\pi}$ **(C)** $\frac{16}{43\pi}$ **(D)** $\frac{9\sqrt{\pi}}{28\pi}$ **(E)** NOTA

18. Let $A = \int_0^1 f(2x)\,dx$ and $B = \int_0^2 f(x)\,dx$, where $f(x)$ is integrable and always positive. Compare A and B.

(A) $A < B$ **(B)** $A = B$ **(C)** $A > B$ **(D)** Depends on $f(x)$ **(E)** NOTA

19. Evaluate:
$$\int_2^4 \frac{3}{x^2-1}\,dx$$

(A) $\frac{3}{2}\ln\left(\frac{9}{5}\right)$ **(B)** $\frac{3}{2}\ln\left(\frac{1}{5}\right)$ **(C)** $\frac{2}{3}\ln\left(\frac{6}{5}\right)$ **(D)** $\frac{1}{3}\ln\left(\frac{1}{5}\right)$ **(E)** NOTA

20. What is the slope of the tangent line to the graph of $3x^3y + 2y^2 = 6xy - x^2$ at the point $(1, 1)$?

(A) -5 **(B)** 0 **(C)** 2 **(D)** 4 **(E)** NOTA

21. Sam the dog likes apples so Mr. Kiser finds an apple with cross-sections given by the polar curve $r(\theta) = 4 - 8\cos(\theta)$. What is the area of the inner-loop of $r(\theta)$?

(A) $\pi + \frac{3\sqrt{3}}{2}$ **(B)** $16\pi - 24\sqrt{3}$ **(C)** $32\pi - 48\sqrt{3}$ **(D)** $32\pi + 48\sqrt{3}$ **(E)** NOTA

22. Evaluate:
$$\lim_{n\to\infty} \frac{\sqrt{n^6 + 4n^5 + 4n^4 + 8n^3 + 3n^2 + n + 7}}{n^3}$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** Does Not Exist **(E)** NOTA

23. Let $x_1, x_2, \ldots$ be an infinite sequence of positive real numbers such that $\sum_{k=1}^{\infty} x_k$ converges. Does $\sum_{k=1}^{\infty} (e^{x_k} - 1)$ converge? (Hint: It follows from convergence that $x_k \to 0$.)

(A) Yes, it must **(B)** No, it can not **(C)** Depends on the x_i **(D)** Need More Info. **(E)** NOTA

24. Evaluate:
$$\lim_{t\to 0} \frac{\cos^2\left(\frac{\pi}{3} + 2t\right) + \sin^2\left(\frac{\pi}{3} - 2t\right) - 1}{2t}$$

(A) $-\infty$ **(B)** $-\frac{\sqrt{3}}{2}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** ∞ **(E)** NOTA

25. Find the length of the curve of the function $f(x) = \frac{2}{3}(x^2-1)^{\frac{3}{2}}$ from $x = 1$ to $x = 3$.

(A) $\frac{19}{3}$ **(B)** $\frac{28}{3}$ **(C)** $\frac{46}{3}$ **(D)** $\frac{58}{3}$ **(E)** NOTA

26. Evaluate: $\int_0^{\infty} xe^{-x^2}\,dx$.

(A) $-\frac{1}{2}$ **(B)** 0 **(C)** $\frac{1}{2}$ **(D)** ∞ **(E)** NOTA

27. Suppose that the vertical and horizontal position functions of a moving particle can be represented by $y(t) = 3\sin(2t) + 5$ and $x(t) = 4\cos(3t) + 8t$, respectively. Find the ratio of the horizontal speed of the particle to the vertical speed of the particle at $t = \frac{\pi}{6}$.

(A) $-\frac{3}{4}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{4}$ **(D)** Need More Info. **(E)** NOTA

28. Evaluate:

$$\sum_{n=1}^{\infty} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^{2n}(x)\, dx$$

(A) $\frac{\pi}{2} - 1$ **(B)** $\frac{\pi}{4} - 2$ **(C)** $1 - \frac{\pi}{4}$ **(D)** $1 + \frac{\pi}{2}$ **(E)** NOTA

29. How many of the following statements regarding sequences and series are true?

I. The coefficient of every even powered term of the Taylor series representation of an even function is 0.

II. A sequence is a list while a series is a numerical value.

III. The sum of the n roots of unity for $z^n = 1$ for all positive integers n is 0.

IV. The interval of convergence for $\sum_{n=1}^{\infty} \frac{x^n}{3^{2n+1}}$ is $[-9, -9]$.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA

30. Kevin likes number theory. Thanks to Euler, he knows that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. Given that $\sum_{n=1}^{\infty} \frac{1}{2^n \cdot n^2}$ can be expressed as $\frac{A\pi^2 + B(\ln 2)^2}{C}$, such that A, B, and C are relatively prime, what is the value of $A + B + C$?

(A) 7 **(B)** 13 **(C)** 19 **(D)** 21 **(E)** NOTA